• The exam consists of 5 questions of equal weight.

• Please read the questions carefully.

• Show all your work in legibly written, well-organized mathematical sentences.

• Calculators and dictionaries are not allowed.

• Simplify as far as possible unless otherwise stated.

• Please turn off your cellular phones before the exam starts.

Please do not write below this line.

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1.a) Prove that

$$\lim_{(x,y) \to (0,0)} (x + y) \sin(x/y) = 0$$
1.b) Find the limit, or show that it does not exist.

\[ \lim_{(x,y) \to (0,0)} \frac{xy^2}{yax^2 + y^4} \]
2.a) Let \( z = yf(x^2 - y^2) \), where \( f \) is a differentiable function. Show that

\[
y^2 z_x + xy z_y = xz
\]
2.b) Find first partial derivatives of \( z \) defined through the equation

\[ e^{z-1}x^2 + xy^2z = 2 \]

at the point \((1, 1, 1)\).
3.a) Let $f(x, y) = xy + y \cos x$. Find the linear approximation of $f$ at the origin.
3.b) (continuation of the previous problem 3.a) Let $R$ be region in $\mathbb{R}^2$ given by

$$R : |x| \leq 0.1 \quad |y| \leq 0.1$$

Find an upper bound for the magnitude of the error in the linear approximation of $f(x, y)$, in $R$. 

4.a) Let \( S_1 : z = x^2 - y^2 \) and \( S_2 : xyz + 30 = 0 \) be two surfaces in \( \mathbb{R}^3 \). Find the equation of the tangent line of the intersection curve of these two surfaces at the point \((-3, 2, 5)\).
4.b) Let $f$ be a function of two variables $x$ and $y$ and $P$ a point in $\mathbb{R}^2$. Derivatives of $f$ in the directions $\vec{u} = \frac{1}{\sqrt{5}}(1, 2)$ and $\vec{v} = \frac{1}{5}(-3, 4)$ are given as

$$(D_u f)_P = \sqrt{5}, \quad (D_v f)_P = 1$$

Find the derivative of $f$ in the direction of $\vec{w} = \frac{1}{\sqrt{2}}(1, -1)$ at $P$. 
5) Let \( f(x, y) = x^2 - 3y^2 + 2xy + x + 3 \), and a triangular region \( R \) is bounded by the lines \( x = 0 \), \( y = 0 \) and \( y + x + 5 = 0 \). Find the absolute maximum and absolute minimum values of the function taken on region \( R \). Find also the saddle points (if any).